Dark energy due to effective quantum field theory

Michael Maziashvili*
Andronikashvili Institute of Physics, 6 Tamarashvili St., Tbilisi 0177, Georgia

In the cosmological context an effective quantum field theory describing the behavior of visible matter in the universe is characterized with its inherent UV cutoff and also with an IR scale that is set by the cosmological (particle) horizon. This UV - IR relation naturally defines a space-time grid over a horizon scale. Using the approach for determining of dark energy through the space-time uncertainty relation versus such a space-time grid, we estimate the energy density and pressure of a dark energy defined by this UV - IR relation. Such a dark energy shows up to decay linearly with time and exhibits a negative pressure only recently.

PACS numbers: 04.60.-m, 06.20.Dk, 95.36.+x, 98.80.-k

Introduction

Having so successful theories of ordinary matter after the study of all these centuries, in 1998 astronomers informed us that ordinary matter constitutes only about 5 percent of the whole mass of the universe [1]. About 70 percent of the mass of the universe is made of what we call now dark energy and about 25 percent of dark matter. Hitherto very little is known about the dark matter, and even less about the dark energy. What we see are their global gravitational effects. They neither emit nor absorb light to any significant extent and generically they seem to interact very feebly not only with photons, but with ordinary matter altogether. Dark energy, because of which the expansion of the universe has recently begun to accelerate, is equally dense everywhere (as far as we can tell) as if it is an intrinsic property of space-time itself. It was noticed long ago by Zeldovich that one of the possible sources for dark energy might be QFT vacuum energy [2]. Namely, as the QFT respects Lorentz invariance, the vacuum energy mimics the cosmological constant, to wit

$$\langle 0|T_{\mu\nu}|0\rangle = \langle 0|T_{00}|0\rangle\,\eta_{\mu\nu}\ , \tag{1}$$

where $\eta_{\mu\nu}$ is a Lorentz metric $\eta_{00} = -\eta_{11} = -\eta_{22} = -\eta_{33} = 1$. Unfortunately vacuum energy density defined as a `Nullpunktsenergie` appears to be infinite. However, the infinity arises from the contribution of modes with very small wavelengths and for we do not know what actually might happen at such scales it is reasonable to introduce a cutoff and hope that a more complete theory will eventually provide a physical justification for doing so. But this is not all the story, as in QFT the energy-momentum operator $T_{\mu\nu}$ (and correspondingly the source of gravity $\langle 0|T_{\mu\nu}|0\rangle$) is not uniquely defined because of operator ordering. In the framework of QFT we are usually subtracting this (divergent) vacuum energy which is equivalent to the normal operator ordering in $T_{\mu\nu}$. Or equivalently in the path integral approach one observes that the equations of motion for matter fields are invariant under the shift of the matter Lagrangian by a constant that results in a new energy momentum tensor

$$T_{\mu\nu} \quad\to\quad T_{\mu\nu} + \text{const.}\,\eta_{\mu\nu}\ .$$

Thus we need some physical principle that could guide us into this problem to guess what is the interplay between QFT and dark energy. Motivated by the definition of dark energy through the space-time uncertainty relation (STU) [3], we consider similar way for defining the vacuum energy density for QFT describing the behavior of matter at different stages of the cosmological evolution.

Energy budget of space-time due to STU

STU for a given background space implies a principal limitation on the space-time distance measurement in light of quantum mechanics and general relativity. The physical meaning of this sort of relation is that during the measurement of some length scale we are disturbing the background space-time, and in view of quantum mechanics and general relativity it turns out that this disturbance can not be reduced arbitrarily. (Throughout this paper we assume natural system of units $\hbar = c = 1$). Taking the Minkowskian background space, on the quite general grounds we notice that a measuring device (or simply a test body) with zero mean velocity, having the mass m and located within the region δx, is characterized by the gravitating energy

$$E = m + \frac{\delta p^2}{m}\ , \tag{2}$$

where $\delta p \simeq \delta x^{-1}$. The second term in this equation accounts for the quantum fluctuation energy of a measuring device. If we are interested in measurement of some local characteristics of the background space, δx can not be taken arbitrarily large. Therefore minimizing the Eq.(2) with respect to m one gets an unavoidable gravitational disturbance of the background space-time. Combining the Heisenberg uncertainty relations with general relativity, Károlyházy obtained STU for Minkowski space of

*Electronic address: maziashvili@iphac.ge

the form [4]

$$\delta t \gtrsim t_P^{2/3} t^{1/3} \ , \tag{3}$$

which tells us that one can not measure the space-time distance t in Minkowski space to a better accuracy than δt. This relation was studied further from different points of view in [3, 5]. STU naturally translates into the metric fluctuations, for if it was possible to measure the background metric precisely one could estimate the length between two points exactly. As we are dealing with the Minkowskian background space the rate of metric fluctuations over a length scale t can be simply estimated as $\delta g_{\mu\nu} \sim \delta t/t$. We naturally expect there to be some energy density associated with these fluctuations. One can use the following simple reasoning for estimating the energy budget of Minkowski space [3]. With respect to the STU relation a length scale t can be known maximum with a precision δt determining thereby a minimal detectable cell δt^3 over a spatial region t^3. Such a cell represents a minimal detectable unit of space-time over a region t^3 and if it has a finite age, t, its existence due to time energy uncertainty relation can not be justified with energy smaller then t^{-1}. Hence, having the STU relation one concludes that if the age of Minkowski space is t then over a spatial region with linear size t (determining the maximal observable patch) there exists a minimal cell δt^3 the energy of which due to time-energy uncertainty relation can not be smaller than t^{-1} leading to

$$\rho_{STU} \sim \frac{1}{t\delta t^3} \ .$$

Using the Eq.(3) one gets

$$\rho_{STU} \sim \frac{1}{t_P^2 t^2} \ , \tag{4}$$

which for $t_0 \sim H_0^{-1}$ gives pretty good value for the present dark energy density. Recently such a parametrization of dark energy by the age of the universe was studied in much details [6]. Two major problems one may expect in such models are as follows. The radiation and matter behave also as $\sim t^{-2}$, correspondingly the present coincidence of the dark energy density with the matter density makes it obscure why such dark energy should become dominant, for instance one may expect its pressure like the matter to be zero [8], and for the same reason it may be hard to reconcile such dark energy models with the early cosmology [9]. So on the quite general grounds one finds that it may be troublesome to avoid these problems in more or less natural way. Nevertheless, it should be noticed that the parametrization of this kind of dark energy by the conformal time allows one to avoid those problems [7].

STU versus the UV - IR relations in QFT

Imposing gravitational bounds on an effective quantum field theory one arrives at the relations between UV and IR scales [10], which on the other hand can be viewed as a space-time uncertainty relations coming from a space-time measurement [11].

For an effective quantum field theory in a box of size l with UV cutoff Λ the entropy S scales as,

$$S \sim l^3 \Lambda^3 \ .$$

That is, the effective quantum field theory counts the degrees of freedom simply as the numbers of cells Λ^{-3} in the box l^3. Nevertheless, considerations involving black holes demonstrate that the maximum entropy in a box of volume l^3 grows only as the area of the box [12]

$$S_{BH} \simeq \left(\frac{l}{l_P}\right)^2 \ .$$

So that, with respect to the Bekenstein bound [12] the degrees of freedom in the volume should be counted by the number of surface cells l_P^2. A consistent physical picture can be constructed by imposing a relationship between UV and IR cutoffs [10]

$$l^3 \Lambda^3 \lesssim S_{BH} \simeq \left(\frac{l}{l_P}\right)^2 \ , \tag{5}$$

where S_{BH} is the entropy of a black hole of size l. Consequently one arrives at the conclusion that the length l, which serves as an IR cutoff, cannot be chosen independently of the UV cutoff, and scales as Λ^{-3}. Rewriting this relation wholly in length terms, $\delta l \equiv \Lambda^{-1}$, one arrives at the Eq.(3).

Another space-time uncertainty relation is based on the random walk approach to the distance measurement, see [13] and the last paper in [11]. Gravitational field is described in terms of space-time metric, so figuratively speaking it measures space-time distances. To measure the space-time distance gravitational field has the only intrinsic length scale l_P. If we assume our ruler is just l_P, that is, both its length and precision are given by the Planck length, we arrive at the equation

$$\delta l \gtrsim l_P \left(\frac{l}{l_P}\right)^{1/2} = l_P^{1/2} l^{1/2} \ . \tag{6}$$

An effective quantum field theory has its own explanation of this relation. In effective quantum field theory the energy density of the vacuum is set by the UV cutoff as $\sim \Lambda^4$. The gravitational radius associated with the vacuum energy of the system, $E_{vacuum} \sim l^3\Lambda^4 \ \Rightarrow \ r_g \sim l_P^2 l^3 \Lambda^4$, will be greater than the size of the system, l, if UV cutoff is defined from the Eq.(5). To be on the safe side, one can impose stronger constraint requiring the size of the system to be greater than the gravitational

radius associated to the maximum energy of the system [10]

$$l_P^2 l^3 \Lambda^4 \lesssim l\,. \tag{7}$$

With respect to this relation the IR cutoff scales like Λ^{-2}. This relation written in length terms ($\delta l \equiv \Lambda^{-1}$) is the Eq.(6).

So, we see the effective quantum field theory picture behind the Eqs.(3, 6). The space-time uncertainty relations, Eqs.(3, 6), in their turn shed new light on the above UV - IR relations (5, 7) obtained in the framework of effective field theory in [10], exhibiting that the IR scale can not be known to a better accuracy than δl representing thereby a lower bound on the admissible UV scale (in length terms).

The main point we want to draw from this section for what follows is that an effective quantum filed theory characterized with UV and IR scales defines a space-time grid that can be considered versus the space-time uncertainty relation.

Defining dark energy due to QFT

In most QFT applications with an UV cutoff it is customary to set the vacuum energy density (simply on the dimensional grounds) as $\sim \Lambda^4$. This energy density is understood to come from `Nullpunktsenergie`. In this regard two remarks are in order. First and main remark as it was discussed in the introduction is that usually in the framework of QFT the vacuum energy $H|0\rangle = E_0|0\rangle$ is treated as an unphysical quantity that may be set arbitrarily[1]. The second remark is more technical and has to do with the regularization procedure. Namely estimating the `Nullpunktsenergie` of QFT in the Minkowskian background, we should care the Eq.(1) to be satisfied [15]. Regularizations of the `Nullpunktsenergie` which respect the Lorentz symmetry of the underling theory disfavor its quartic dependence on the UV scale, but rather it appears to depend quadratically on the UV scale [15]. This point has attracted little attention hitherto for many authors still follow the old customary. One may object that the presence of a IR scale immediately brakes the Lorentz invariance and through the box boundary conditions naturally leads to this estimate of `Nullpunktsenergie` (as we did in the previous section). But we should recall that in the cosmological context IR scale set by the particle horizon defines merely a causally connected region and does not imply any box boundary conditions at this scale.

In what follows we will skip this `Nullpunktsenergie` paradigm.

The behavior of matter in the universe at different stages of its evolution is described by the particle physics models, which in the framework of an effective quantum field theory are characterized with their intrinsic UV energy scales [16]. The microscopic energy scales of quantum mechanics and the macroscopic properties of our present Universe are intimately connected. For instance, the O(eV) energy scale of atomic physics manifests itself through the existence of the cosmic microwave background radiation, and the O(MeV) scale of nuclear physics through the primordial origin of light element abundances. The connection can be extended further. As we move further back in time, several phase transitions in the universe might be available. One can order the sequence of early time cosmological phase transitions roughly as: The GUT phase transition when the universe was about $\sim 10^{-35}$s old and the temperature about $T_{GUT} \sim 10^{16}\,\mathrm{GeV}$;[2] the EW phase transition when the universe was about $\sim 10^{-12}$s old with a temperature $T_{EW} \sim 100\,\mathrm{GeV}$; the QCD phase transition at about $\sim 10^{-5}$s when the temperature was about $T_{QCD} \sim 170\,\mathrm{MeV}$. So that the UV scale suggested by the particle physics describing the behavior of matter in the universe at different stages does not follow neither Eq.(3) nor Eq.(6) (or equivalently neither Eq.(5) nor Eq.(7)). Taking the $\Lambda(t)$ that follows from particle physics models describing different stages of the universe and straightforwardly repeating the discussion of the second section we get[3]

$$\rho_{QFT} \simeq \frac{\Lambda^3(t)}{t}\ . \tag{8}$$

From above discussion one infers that after the nucleosynthesis (which started when the universe was about ~ 1s old) we can take $\Lambda = O(\mathrm{MeV})$. Taking Λ to be about $\Lambda \simeq 100\,\mathrm{MeV}$ after the nucleosynthesis, from Eq.(8) one gets pretty good value for the present dark energy density. As this energy density decays linearly it will not affect the early time cosmology. The equation of state can be simply estimated. Assuming that this energy component dominates

$$H^2 = \frac{8\pi}{3m_P^2}\rho_{QFT}\ ,$$

and using energy-momentum conservation

$$p = -\frac{\dot{\rho}}{3H} - \rho\ ,$$

[1] For a crystal the `Nullpunktsenergie` represents the vibration energy of crystal molecules at a zero temperature. This energy manifests itself even at a finite temperature, and has therefore quite definite physical meaning. One can see for instance very readable popular book by Kaganov [14], where many conceptual points of condensed matter physics are elucidated.

[2] Let us notice that many models of inflation indicate that the universe never had such a high temperature after inflation.

[3] Motivated with Dirac's large number hypothesis similar expression was suggested by Zee in [17].

one gets

$$p_{QFT} \simeq \sqrt{\frac{m_P^2 \Lambda^3}{24\pi t^3}} - \frac{\Lambda^3}{t} \ . \qquad (9)$$

The second term in Eq.(9) becomes dominant for

$$t \gtrsim \frac{m_P^2}{24\pi \Lambda^3} \simeq 10^{58} t_P \ . \qquad (10)$$

So this dark energy exhibits a negative pressure just recently.

Concluding remarks

In the cosmological context we are operating with two length scales, the IR and UV ones, where IR scale is naturally set by the horizon while UV scale is determined by the effective quantum field theory describing the behavior of matter in the universe. These two length scales uniquely define a space-time grid over a causally connected region, or simply a causally connected space-time grid that can be studied versus the space-time uncertainty relation. Namely, assuming the finiteness of the age of a space-time, t, we conclude that due to time-energy uncertainty relation the spatial cell Λ^{-3} set by the UV scale over the observable region can not be smaller than t^{-1}, that immediately leads to the Eq.(8). After the QCD phase transition in the universe we can take the UV scale to be in the O(MeV) range. Taking $\Lambda \simeq 100$MeV after the quark confinement transition in the universe, one gets pretty good value for present dark energy density and also that value of the UV scale guaranties negative pressure at the present stage, Eqs.(9, 10), as befits a dark energy. The advantage of the QFT dark energy model, Eq.(8), over the STU dark energy, Eq.(4), is that as it decays linearly with time it can not spoil the successes of early cosmology and, on the other hand, it obviously exhibits a negative pressure for the present cosmological stage. As an interesting feature the pressure of this dark energy, Eq.(9), becomes negative only recently, Eq.(10). So that the early time cosmology is doubly protected from the action of this dark energy. Certainly the validity of this sort of dark energy requires further detailed analysis against the observed cosmological data.

Author is greatly indebted to Professor Alexander Vilenkin for invitation and hospitality at the *Tufts Institute of Cosmology*, where this paper was done. Author is also indebted to Professors Rong-Gen Cai and Naoki Sasakura for stimulating comments. This work was supported by the *CRDF/GRDF*, the *INTAS Fellowship for Young Scientists* and the *Georgian President Fellowship for Young Scientists*.

---

[1] S. Perlmutter et al., Astrophys. J. 517 (1999) 565, astro-ph/9812133; A. G. Riess, et al., Astron. J. (1998) 116, 1009, astro-ph/9805201.
[2] Y. B. Zeldovich, Pisma Zh. Eksp. Teor. Fiz. 6 (1967) 883; Usp. Fiz. Nauk 95 (1968) 209.
[3] N. Sasakura, Prog. Theor. Phys. 102 (1999) 169, hep-th/9903146; M. Maziashvili, Int. J. Mod. Phys. D16 (2007) 1531, gr-qc/0612110.
[4] F. Károlyházy, Nuovo Cim. A42 (1966) 390; F. Károlyházy, A. Frenkel and B. Lukács, in *Physics as Natural Philosophy* (Eds. A. Shimony and H. Feschbach, MIT Press, Cambridge, MA, 1982); F. Károlyházy, A. Frenkel and B. Lukács, in *Quantum Concepts in Space and Time* (Eds. R. Penrose and C. J. Isham, Clarendon Press, Oxford, 1986).
[5] Y. J. Ng and H. van Dam, Mod. Phys. Lett. A9 (1994) 335.
[6] R. G. Cai, Phys. Lett. B657 (2007) 228, arXiv: 0707.4049 [hep-th]; H. Wei and R. G. Cai, arXiv: 0707.4052 [hep-th]; H. Wei and R. G. Cai, Phys. Lett. B655 (2007) 1, arXiv: 0707.4526 [physics.gen-ph]; X. Wu, Y. Zhang, H. Li, R. G. Cai and Z. H. Zhu, arXiv: 0708.0349 [astro-ph]; Y. Zhang, H. Li, X. Wu, H. Wei and R. G. Cai, arXiv: 0708.1214 [astro-ph]; I. P. Neupane, arXiv: 0708.2910 [hep-th]; K. Y. Kim, H. W. Lee and Y. S. Myung, arXiv: 0709.2743 [gr-qc]; I. P. Neupane, arXiv: 0709.3096 [hep-th].
[7] H. Wei and R. G. Cai, Phys. Lett. B660 (2008) 113, arXiv: 0708.0884 [astro-ph]; arXiv: 0708.1894 [astro-ph].
[8] S. D.H. Hsu, Phys. Lett. B594 (2004) 13, hep-th/0403052.
[9] T. Padmanabhan, Class. Quant. Grav. 19 (2002) L167, gr-qc/0204020; Class. Quant. Grav. 19 (2002) 3551, gr-qc/0110046; J. D. Barrow, Phys. Rev. D75 (2007) 067301, gr-qc/0612128.
[10] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Rev. Lett. 82 (1999) 4971, hep-th/9803132.
[11] M. Maziashvili, Phys. Lett. B652 (2007) 165, arXiv: 0705.0924 [gr-qc]; arXiv: 0708.1472 [hep-th]; arXiv: 0709.0898 [gr-qc].
[12] J. D. Bekenstein, Phys. Rev. D7 (1973) 2333; Phys. Rev. D9 (1974) 3292; Phys. Rev. D23 (1981) 287; Phys. Rev. D49 (1994) 1912, gr-qc/9307035.
[13] G. Amelino-Camelia, Lect. Notes Phys. 541 (2000) 1, gr-qc/9910089.
[14] M. I. Kaganov, *Electrons, Phonons, Magnons*, (Moscow, Nauka, 1979).
[15] E. Kh. Akhmedov, hep-th/0204048; G. Ossola and A. Sirlin, Eur. Phys. J. C31 (2003) 165, hep-ph/0305050.
[16] V. Rubakov, *Introduction to cosmology*, PoS (RTN2005) 003.
[17] A. Zee, in *High Energy Physics: in Honor of P. A. M. Dirac in his Eighties Year* (Eds. S. L. Mintz and A. Perlmutter, Plenum Press, New York, 1985); Mod. Phys. Lett. A19 (2004) 983, hep-th/0403064.